On the Whirling of Rotating Shaft at High Rotational Speed

AUTHOR(S):

Kokame, Zyun; Katase, Akira; Yano, Syukuro

2. On the Whirling of Rotating Shaft at High Rotational Speed.

Zyun Kokame, Akira Katase and *Syukuro Yano.*

It is well known that the rotating shaft or bar often becomes unstable and finally bends double or breaks down at a rotational speed near its natural frequency of lateral vibration, on account of so called whirling phenomena. In our experiment, the magnetically freely suspended rod[1], 2.277 cm in length, 0.298 cm in diameter, bent double at the rotational frequency of 2.49×10^4 r. p. s. which is considerably lower than the expected value from ordinary calculations. Taking the shearing strain of the rod into consideration, which is ordinarily ignored, we tried to perform more accurate calculation, and obtained the following equation for an elastic lateral vibration of the bar rotating with angular velocity ω,

$$\frac{Ek^2}{\rho}\frac{\partial^4 y}{\partial x^4}+\left(1-\frac{k^2\rho\omega^2}{\lambda G}\right)\frac{\partial^2 y}{\partial t^2}-k^2\left(1+\frac{E}{\lambda G}\right)\frac{\partial^4 y}{\partial x^2\partial t^2}+\frac{k^2}{\lambda G}\frac{\partial^4 y}{\partial t^4}+\omega^2\left(k^2\frac{E}{\lambda G}\frac{\partial^2 y}{\partial x^2}-y\right)=0 \quad (1)$$

where E is Young's modulus, ρ the density, k the radius of gyration of cross-section about an axis perpendicular to the rod axis, G the rigidity, λ the constant concerning the shape of cross-section. This can be solved easily so far as only the threshold frequency of whirling is in question, i. e., the case $\frac{\partial^2 y}{\partial t^2}=0$. Then Eq. (1) is simplified to

$$\frac{Ek^2}{\rho}\frac{d^4 y}{dx^4}+\omega^2\varepsilon k^2\frac{d^2 y}{dx^2}-\omega^2 y=0, \quad \varepsilon=\frac{E}{\lambda G} \quad (2)$$

Integrating Eq. (2) after the same method as in ordinary Eigen value problem, we obtain the value of ω at which the bar becomes unstable. The result is

$$\omega=k\frac{z^2}{l^2}\sqrt{\frac{E}{\rho}} \quad (3)$$

Where,

$$z^4=\frac{47520}{L}\left(N-\sqrt{N^2-\frac{7}{198}L}\right)$$

$$L=64+8448\beta^2\varepsilon+95040\beta^4\varepsilon^2+\ldots\ldots\ldots\ldots$$

$$N=2+56\beta^2\varepsilon-210\beta^4\varepsilon^2+\ldots\ldots\ldots\ldots$$

and l is the length of the bar. In our case, $E=20.64\times10^{11}$dyne/cm², $G=8.07\times10^{11}$dyne/cm², $\rho=7.83$, $k=0.0747$ cm, $\varepsilon=2.76$. so that the Eq. (3) gives the value $f=\frac{\omega}{2\pi}=2.54\times10^4$ r. p. s. which is yet slightly higher than experimental value. With a few experimental data obtained so far, it is not yet possible to decide whether or not this disagreement depends on the extraordinary behaviour of elastic body in a very high centrifugal field. More precise investigations are in progress both experimentally and theoretically.

1) Cf. the previous report.

References:

S. P. Timoshenko, Phil. Mag. **41**, 744 ('21)

R. M. Davies, Phil. Mag. **22**, 872 ('36); **23**, 464, 563, 1129('37)